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The $g_{\Sigma_Q\Sigma_Q\pi}$ Coupling Constant via Light Cone QCD Sum Rules

K. Azizi[1*], M. Bayar[2§], A. Ozpineci[3†], Y. Sarac[4‡]

[1] *Physics Division, Faculty of Arts and Sciences, Doğuş University*
[*] *kazizi@dogus.edu.tr*

[2] *Department of Physics,*
Kocaeli University, 41380 Izmit, Turkey
[§] *melahat.bayar@kocaeli.edu.tr*

[3] *Physics Department, Middle East Technical University,*
06531, Ankara, Turkey
[†] *ozpineci@metu.edu.tr*

[4] *Electrical and Electronics Engineering Department,*
Atilim University, 06836 Ankara, Turkey
[‡] *ysoymak@atilim.edu.tr*

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Using the most general form of the interpolating currents, the coupling constants $g_{\Sigma_b\Sigma_b\pi}$ and $g_{\Sigma_c\Sigma_c\pi}$ are calculated within the light cone QCD sum rules approach. It is found that $g_{\Sigma_c\Sigma_c\pi} = -8.0 \pm 1.7$ and $g_{\Sigma_b\Sigma_b\pi} = -11.0 \pm 2.1$.

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I. INTRODUCTION

Theoretical studies on heavy baryons containing a single heavy quark have gained pace recently as a result of the experimental progresses in the last few years (see [1] and references therein). The presence of a heavy b or c quark in these baryons makes them more attractive, theoretically. They can provide information on the structure of the QCD and its parameters as well as clues on new physics effects. For this reason, it is important to understand their properties as precisely as possible. Couplings of these baryons to pions are one of the important properties, also because their pion cloud can significantly modify their properties.

In light of the experimental developments, the mass spectrum of these baryons has been studied extensively via QCD sum rules method both in the finite mass limit (see e.g. [2]) and heavy quark limit [3], in the heavy quark effective theory[4] and different phenomenological models (see for example [5–10]). Electromagnetic [11–17] and strong [18–21] interactions are two of other important characteristic of hadrons that can be used to probe their structures.

In the present work, the coupling of the Σ_Q baryons, $Q = c$ or b, to pion is calculated in the light cone QCD sum rules (LCQSR) framework. This problem has been studied using chiral perturbation theory in [21]. Note that, the same framework has also been used to calculate the strong coupling constants of the light mesons with heavy baryons [18–20]. The layout of the paper is as follows: in the next section, we obtain sum rules for the corresponding coupling constants in the framework of LCQSR. Section 3 contains our numerical analysis and discussion.

II. LIGHT CONE QCD SUM RULES FOR THE $g_{\Sigma_Q\Sigma_Q\pi}$ COUPLING CONSTANT

To study the coupling constant in LCQSR method, the following correlation function is studied:

$$\Pi = i \int d^4x e^{ipx} \langle \pi(q) \mid \mathcal{T}\{\eta_{\Sigma_Q^{ud}}(x)\bar{\eta}_{\Sigma_Q^{uu}}(0)\} \mid 0\rangle, \quad (1)$$

where $\eta_{\Sigma_Q^{q_1q_2}}(x)$ is the interpolating current of $\Sigma_Q^{q_1q_2}$ baryon containing the quarks q_1q_2Q ($q_i = u,$ or d) and $\mathcal{T}$ stands for the time ordered product. The correlator can be calculated in terms of hadronic parameters inserting a complete set of hadronic states. This expression is called the phenomenological or physical side. It can also be calculated in

theoretical side in terms of QCD parameters via operator product expansion (OPE) in deep Euclidean region, where $-p^2 \to \infty$ and $-(p+q)^2 \to \infty$. QCD sum rules for the considered coupling constant is obtained matching both sides of the correlation function and applying Borel transformation in order to suppress contribution of the higher states and continuum.

First, we focus our attention to calculate the physical side of the correlation function. The phenomenological side of the sum rules for the heavy baryons is similar to the light baryons [22]. For this aim, we insert a complete set of hadronic state having the same quantum numbers of the interpolating currents into the correlator. As a result, we obtain

$$\Pi = \frac{\langle 0 \mid \eta_{\Sigma_Q} \mid \Sigma_Q(p_2)\rangle}{p_2^2 - m_{\Sigma_Q}^2} \langle \Sigma_Q(p_2)\pi(q) \mid \Sigma_Q(p_1)\rangle \frac{\langle \Sigma_Q(p_1) \mid \bar{\eta}_{\Sigma_Q} \mid 0\rangle}{p_1^2 - m_{\Sigma_Q}^2} + ..., \tag{2}$$

where $p_1 = p+q$ and $p_2 = p$ and ... denotes the contribution of the higher states and continuum. The matrix element creating baryon from the vacuum is parameterized as:

$$\langle 0 \mid \eta_{\Sigma_Q} \mid \Sigma_Q(p,s)\rangle = \lambda_{\Sigma_Q} u_{\Sigma_Q}(p,s), \tag{3}$$

where λ_{Σ_Q} is the residue and u_{Σ_Q} is the spinor for the Σ_Q baryon. The remaining matrix element appearing in Eq. (2) defines the coupling constant $g_{\Sigma_Q\Sigma_Q\pi}$ through

$$\langle \Sigma_Q(p_2)\pi(q) \mid \Sigma_Q(p_1)\rangle = g_{\Sigma_Q\Sigma_Q\pi}\overline{u}(p_2) i\gamma_5 u(p_1). \tag{4}$$

Using the Eqs. (3) and (4) in Eq. (2) we obtain the phenomenological side of the correlator as:

$$\Pi = i \frac{g_{\Sigma_Q\Sigma_Q\pi}|\lambda_{\Sigma_Q}|^2}{(p_1^2 - m_{\Sigma_Q}^2)(p_2^2 - m_{\Sigma_Q}^2)} \left[- \not{p} \not{q}\gamma_5 - m_{\Sigma_Q} \not{q}\gamma_5 \right]. \tag{5}$$

In principle, one can choose any of the structures, $\not{p} \not{q}\gamma_5$ and $\not{q}\gamma_5$, existing in Eq. (5). Our calculations show that the structure $\not{p} \not{q}\gamma_5$ leads to a more reliable result.

After obtaining the physical side, we proceed to calculate the QCD side of the correlation function. For this purpose, we use the interpolating currents in the following general form:

$$\begin{aligned}
\eta_{\Sigma_Q^{ud}} &= \frac{1}{\sqrt{2}}\epsilon^{abc}\Bigg[\Big(u_a^T C Q_b\Big)\gamma^5 d_c + \beta\Big(u_a^T C\gamma^5 Q_b\Big) d_c - \Big(Q_a^T C d_b\Big)\gamma^5 u_c - \beta\Big(Q_a^T C\gamma^5 d_b\Big) u_c\Bigg], \\
\eta_{\Sigma_Q^{uu}} &= -\frac{1}{2}\epsilon^{abc}\Bigg[\Big(u_a^T C Q_b\Big)\gamma^5 u_c + \beta\Big(u_a^T C\gamma^5 Q_b\Big) u_c - \Big(Q_a^T C u_b\Big)\gamma^5 u_c - \beta\Big(Q_a^T C\gamma^5 u_b\Big) u_c\Bigg].
\end{aligned} \tag{6}$$

In Eq. (6) the Q denotes the heavy quarks b or c, β is an arbitrary parameter with $\beta = -1$ corresponding to the Ioffe current, C is the charge conjugation operator and a, b and c represent the color indices.

To proceed in our calculations in QCD side, we need also the propagators of the heavy and light quarks. They are given as [23]:

$$\begin{aligned}
S_Q(x) &= S_Q^{free}(x) - ig_s \int \frac{d^4k}{(2\pi)^4} e^{-ikx} \int_0^1 dv \left[\frac{\not{k} + m_Q}{(m_Q^2 - k^2)^2} G^{\mu\nu}(vx)\sigma_{\mu\nu} + \frac{1}{m_Q^2 - k^2} v x_\mu G^{\mu\nu}\gamma_\nu\right], \\
S_q(x) &= S_q^{free}(x) - \frac{\langle\bar{q}q\rangle}{12} - \frac{x^2}{192} m_0^2 \langle\bar{q}q\rangle \\
&\quad - ig_s \int_0^1 du \left[\frac{\not{x}}{16\pi^2 x^2} G_{\mu\nu}(ux)\sigma_{\mu\nu} - u x^\mu G_{\mu\nu}(ux)\gamma^\nu \frac{i}{4\pi^2 x^2}\right].
\end{aligned} \tag{7}$$

The free light and heavy quark propagators in Eq. (7) are given in x representation as

$$\begin{aligned}
S_q^{free} &= \frac{i \not{x}}{2\pi^2 x^4}, \\
S_Q^{free} &= \frac{m_Q^2}{4\pi^2} \frac{K_1(m_Q\sqrt{-x^2})}{\sqrt{-x^2}} - i \frac{m_Q^2 \not{x}}{4\pi^2 x^2} K_2(m_Q\sqrt{-x^2}),
\end{aligned} \tag{8}$$

where K_i are the Bessel functions. Besides the propagators, the matrix elements of the form $\langle\pi(q)|\bar{q}(x_1)\Gamma_i q(x_2)|0\rangle$ are also required. Here Γ_i represents any member of the Dirac basis i.e., $\{1, \gamma_\alpha, \sigma_{\alpha\beta}/\sqrt{2}, i\gamma_5\gamma_\alpha, \gamma_5\}$. The matrix elements $\langle\pi(q)|\bar{q}(x_1)\Gamma_i q(x_2)|0\rangle$ are parameterized in terms of the pion light cone distribution amplitudes and they are given explicitly as [24, 25]

$$
\begin{aligned}
\langle\pi(p)|\bar{q}(x)\gamma_\mu\gamma_5 q(0)|0\rangle &= -if_\pi p_\mu \int_0^1 du e^{i\bar{u}px}\left(\varphi_\pi(u) + \frac{1}{16}m_\pi^2 x^2 \mathbb{A}(u)\right) \\
&- \frac{i}{2} f_\pi m_\pi^2 \frac{x_\mu}{px}\int_0^1 du e^{i\bar{u}px}\mathbb{B}(u), \\
\langle\pi(p)|\bar{q}(x) i\gamma_5 q(0)|0\rangle &= \mu_\pi \int_0^1 du e^{i\bar{u}px}\varphi_P(u), \\
\langle\pi(p)|\bar{q}(x)\sigma_{\alpha\beta}\gamma_5 q(0)|0\rangle &= \frac{i}{6}\mu_\pi\left(1-\tilde{\mu}_\pi^2\right)\left(p_\alpha x_\beta - p_\beta x_\alpha\right)\int_0^1 du e^{i\bar{u}px}\varphi_\sigma(u), \\
\langle\pi(p)|\bar{q}(x)\sigma_{\mu\nu}\gamma_5 g_s G_{\alpha\beta}(vx) q(0)|0\rangle &= i\mu_\pi\left[p_\alpha p_\mu\left(g_{\nu\beta} - \frac{1}{px}(p_\nu x_\beta + p_\beta x_\nu)\right)\right. \\
&- p_\alpha p_\nu\left(g_{\mu\beta} - \frac{1}{px}(p_\mu x_\beta + p_\beta x_\mu)\right) \\
&- p_\beta p_\mu\left(g_{\nu\alpha} - \frac{1}{px}(p_\nu x_\alpha + p_\alpha x_\nu)\right) \\
&+ \left. p_\beta p_\nu\left(g_{\mu\alpha} - \frac{1}{px}(p_\mu x_\alpha + p_\alpha x_\mu)\right)\right] \\
&\times \int \mathcal{D}\alpha e^{i(\alpha_{\bar{q}} + v\alpha_g)px}\mathcal{T}(\alpha_i), \\
\langle\pi(p)|\bar{q}(x)\gamma_\mu\gamma_5 g_s G_{\alpha\beta}(vx) q(0)|0\rangle &= p_\mu(p_\alpha x_\beta - p_\beta x_\alpha)\frac{1}{px} f_\pi m_\pi^2 \int \mathcal{D}\alpha e^{i(\alpha_{\bar{q}} + v\alpha_g)px}\mathcal{A}_\parallel(\alpha_i) \\
&+ \left[p_\beta\left(g_{\mu\alpha} - \frac{1}{px}(p_\mu x_\alpha + p_\alpha x_\mu)\right)\right. \\
&- \left. p_\alpha\left(g_{\mu\beta} - \frac{1}{px}(p_\mu x_\beta + p_\beta x_\mu)\right)\right] f_\pi m_\pi^2 \\
&\times \int \mathcal{D}\alpha e^{i(\alpha_{\bar{q}} + v\alpha_g)px}\mathcal{A}_\perp(\alpha_i), \\
\langle\pi(p)|\bar{q}(x)\gamma_\mu i g_s G_{\alpha\beta}(vx) q(0)|0\rangle &= p_\mu(p_\alpha x_\beta - p_\beta x_\alpha)\frac{1}{px} f_\pi m_\pi^2 \int \mathcal{D}\alpha e^{i(\alpha_{\bar{q}} + v\alpha_g)px}\mathcal{V}_\parallel(\alpha_i) \\
&+ \left[p_\beta\left(g_{\mu\alpha} - \frac{1}{px}(p_\mu x_\alpha + p_\alpha x_\mu)\right)\right. \\
&- \left. p_\alpha\left(g_{\mu\beta} - \frac{1}{px}(p_\mu x_\beta + p_\beta x_\mu)\right)\right] f_\pi m_\pi^2 \\
&\times \int \mathcal{D}\alpha e^{i(\alpha_{\bar{q}} + v\alpha_g)px}\mathcal{V}_\perp(\alpha_i),
\end{aligned} \tag{9}
$$

where $\mu_\pi = f_\pi \frac{m_\pi^2}{m_u + m_d}$, $\tilde{\mu}_\pi = \frac{m_u + m_d}{m_\pi}$, $\mathcal{D}\alpha = d\alpha_{\bar{q}} d\alpha_q d\alpha_g \delta(1 - \alpha_{\bar{q}} - \alpha_q - \alpha_g)$ and the $\varphi_\pi(u)$, $\mathbb{A}(u)$, $\mathbb{B}(u)$, $\varphi_P(u)$, $\varphi_\sigma(u)$, $\mathcal{T}(\alpha_i)$, $\mathcal{A}_\perp(\alpha_i)$, $\mathcal{A}_\parallel(\alpha_i)$, $\mathcal{V}_\perp(\alpha_i)$ and $\mathcal{V}_\parallel(\alpha_i)$ are functions of definite twist and their expressions will be given in the numerical analysis section. Using these inputs, the QCD side of the correlator can be calculated in a straightforward fashion.

Having calculated the correlation function both in physical and QCD sides, we equate the coefficients of the selected structure from both representations and apply Borel transformation with respect to variables p^2 and $(p+q)^2$ to suppress the contribution of higher states and continuum. As a result of these procedures, the QCD sum rules for coupling constant $g_{\Sigma_Q \Sigma_Q \pi}$ is obtained as:

$$
e^{\frac{-m_{\Sigma_Q}}{M^2}} m_{\Sigma_Q} |\lambda_{\Sigma_Q}|^2 g_{\Sigma_Q \Sigma_Q \pi} = \int_{m_Q^2}^{s_0} e^{-\frac{s}{M^2} - \frac{m_\pi^2}{4M^2}} \rho(s) ds + e^{-\frac{m_Q^2}{M^2} - \frac{m_\pi^2}{4M^2}} \Gamma, \tag{10}
$$

where the functions, $\rho(s)$ and Γ are given as:

$$
\begin{aligned}
\rho(s) \;=\; & -\frac{1}{64\sqrt{2}\pi^2}\Bigg[2(1-\beta)^2 m_Q^3 f_\pi\{2\psi_{10}-\psi_{20}+\psi_{31}-2\ln(\frac{s}{m_Q^2})\}\varphi_\pi(u_0) \\
& -2(1-\beta^2)(\psi_{20}-\psi_{31})m_Q^2(-1+\widetilde{\mu_\pi}^2)\mu_\pi\varphi_\sigma(u_0)+2m_Q(1-\beta)^2(\psi_{10}-\psi_{21})A(u_0)f_\pi m_\pi^2 \\
& +2(1-\beta)\{(1-\beta)m_Q f_\pi[2(\psi_{10}-\psi_{21})(\eta_3+\eta_1-\eta_2)-(\eta_1-\eta_2)\ln(\frac{s}{m_Q^2})]m_\pi^2 \\
& +(1+\beta)[m_Q^2(\eta_4'-\eta_5')(\psi_{10}-\psi_{20}+\psi_{31}-\ln(\frac{s}{m_Q^2}))+4(\psi_{10}-\psi_{11}-\psi_{12}+\psi_{21})u_0(\eta_4'-2\eta_5')m_\pi^2]m_\pi \\
& -2(-1+\beta)m_Q\ln(\frac{s}{m_Q^2})[(1-\beta)(\eta_1-2\eta_2)f_\pi m_\pi^2+(1+\beta)m_Q(\eta_4'-2\eta_5')m_\pi] \\
& +2(-1+\beta)\{2(-1+\beta)(\psi_{10}+\psi_{21})m_Q(\eta_3+\eta_1-\eta_2)f_\pi m_\pi^2+(1+\beta)[(\psi_{10}-\psi_{20}+\psi_{31})m_Q^2(\eta_4'-2\eta_5') \\
& +4(\psi_{10}-\psi_{11}-\psi_{12}+\psi_{21})u_0(\eta_4-2\eta_5)m_\pi^2]m_\pi\}\Bigg]+\frac{\langle\bar{u}u\rangle}{12\sqrt{2}}(-1+\beta^2)\psi_{00}f_\pi\varphi_\pi(u_0),
\end{aligned}
\tag{11}
$$

$$
\begin{aligned}
\Gamma \;=\; & -\frac{\langle\bar{u}u\rangle}{288\sqrt{2}}(6m_Q^2+1)(-1+\beta^2)m_0^2 f_\pi\varphi_\pi(u_0)-\frac{\langle\bar{u}u\rangle}{216\sqrt{2}M^4}\Bigg[(-1+\widetilde{\mu_\pi}^2)m_\pi\Big(6M^4 m_Q(3+2\beta+6\beta^2) \\
& -\frac{3}{2}m_0^2 m_Q^3(3+2\beta+6\beta^2)+M^2 m_0^2 m_Q(5+4\beta+5\beta^2)\Big)\varphi_\sigma(u_0)\Bigg] \\
& -\frac{\langle\bar{u}u\rangle}{1152\sqrt{2}M^6}(1-\beta)^2 f_\pi m_\pi^2\Big[24M^6-6m_0^2m_Q^4+5M^2m_0^2m_Q^2+24M^4m_Q^2\Big]A(u_0) \\
& +\frac{\langle\bar{u}u\rangle}{96\sqrt{2}M^4}(-1+\beta^2)(\eta_3-\eta_1)f_\pi m_\pi^2(m_0^2m_Q^2-4M^4),
\end{aligned}
\tag{12}
$$

where

$$
\begin{aligned}
\eta_j &= \int\mathcal{D}\alpha_i\int_0^1 dv f_j(\alpha_i)\delta(\alpha_{\bar{q}}+(1-v)\alpha_g-u_0), \\
\eta_j' &= \int\mathcal{D}\alpha_i\int_0^1 dv f_j(\alpha_i)\delta'(\alpha_{\bar{q}}+(1-v)\alpha_g-u_0), \\
\psi_{nm} &= \frac{(s-{m_Q}^2)^n}{s^m(m_Q^2)^{n-m}},
\end{aligned}
\tag{13}
$$

and $f_1(\alpha_i)=\mathcal{V}_\parallel(\alpha_i),\ f_2(\alpha_i)=\mathcal{V}_\perp(\alpha_i),\ f_3(\alpha_i)=\mathcal{A}_\parallel(\alpha_i),\ f_4(\alpha_i)=\mathcal{T}(\alpha_i),\ f_5(\alpha_i)=v\mathcal{T}(\alpha_i)$ are the pion distribution amplitudes. Note that, in the above equations, the Borel parameter M^2 is defined as $M^2=\frac{M_1^2M_2^2}{M_1^2+M_2^2}$ and $u_0=\frac{M_1^2}{M_1^2+M_2^2}$. Since the masses of the initial and final baryons are the same, we can set $M_1^2=M_2^2$ and $u_0=\frac{1}{2}$.

As it is clear from the expression of the coupling constant, we need also the residue, λ_{Σ_Q}, which is calculated using mass sum rules [20]:

$$
-\lambda_{\Sigma_Q}^2 e^{-m_{\Sigma_Q}^2/M^2} = \int_{m_Q^2}^{s_0} e^{\frac{-s}{M^2}}\rho_1(s)ds+e^{\frac{-m_Q^2}{M^2}}\Gamma_1, \tag{14}
$$

with

$$
\begin{aligned}
\rho_1(s) \;=\; & (<\overline{d}d>+<\overline{u}u>)\frac{(\beta^2-1)}{64\pi^2}\Bigg\{\frac{m_0^2}{4m_Q}(6\psi_{00}-13\psi_{02}-6\psi_{11}) \\
& +\; 3m_Q(2\psi_{10}-\psi_{11}-\psi_{12}+2\psi_{21})\Bigg\}
\end{aligned}
$$

$$+ \frac{m_Q^4}{2048\pi^4}[5+\beta(2+5\beta)][12\psi_{10} - 6\psi_{20} + 2\psi_{30} - 4\psi_{41} + \psi_{42} - 12ln(\frac{s}{m_Q^2})], \tag{15}$$

$$\Gamma_1 = \frac{(\beta-1)^2}{24} < \bar{d}d >< \bar{u}u > \left[\frac{m_Q^2 m_0^2}{2M^4} + \frac{m_0^2}{4M^2} - 1 \right], \tag{16}$$

Note that, since only the square of the residue appears in Eq. (14), only the magnitude (not the sign) of λ_{Σ_Q} can be determined from mass sum rules. This indeterminacy is also carried into the coupling constant calculation, and hence sum rules determine the coupling constant upto a sign.

III. NUMERICAL ANALYSIS

This section is devoted to the numerical analysis of the coupling constant $g_{\Sigma_Q\Sigma_Q\pi}$. The numerical values of the required input parameters are given as: $\langle \bar{q}q\rangle(1\ GeV) = -(0.243)^3\ GeV^3$, $m_b = 4.7\ GeV$, $m_c = 1.27\ GeV$, $m_{\Sigma_b} = 5.805\ GeV$,$m_{\Sigma_c} = 2.455\ GeV$, $m_0^2(1\ GeV) = (0.8 \pm 0.2)\ GeV^2$ [27], $f_\pi = 0.131$ [24, 27] and $m_\pi = 0.135\ GeV$. In order to calculate the coupling constant, the π-meson wave functions are needed and explicit forms of the these wave functions are represented as [24, 25]

$$\begin{aligned}
\phi_\pi(u) &= 6u\bar{u}\left(1 + a_1^\pi C_1(2u-1) + a_2^\pi C_2^{\frac{3}{2}}(2u-1)\right),\\
\mathcal{T}(\alpha_i) &= 360\eta_3\alpha_{\bar{q}}\alpha_q\alpha_g^2\left(1 + w_3\frac{1}{2}(7\alpha_g - 3)\right),\\
\phi_P(u) &= 1 + \left(30\eta_3 - \frac{5}{2}\mu_\pi^2\right)C_2^{\frac{1}{2}}(2u-1)\\
&+ \left(-3\eta_3 w_3 - \frac{27}{20}\mu_\pi^2 - \frac{81}{10}\mu_\pi^2 a_2^\pi\right)C_4^{\frac{1}{2}}(2u-1),\\
\phi_\sigma(u) &= 6u\bar{u}\left[1 + \left(5\eta_3 - \frac{1}{2}\eta_3 w_3 - \frac{7}{20}\mu_\pi^2 - \frac{3}{5}\mu_\pi^2 a_2^\pi\right)C_2^{\frac{3}{2}}(2u-1)\right],\\
\mathcal{V}_\parallel(\alpha_i) &= 120\alpha_q\alpha_{\bar{q}}\alpha_g\left(v_{00} + v_{10}(3\alpha_g - 1)\right),\\
\mathcal{A}_\parallel(\alpha_i) &= 120\alpha_q\alpha_{\bar{q}}\alpha_g\left(0 + a_{10}(\alpha_q - \alpha_{\bar{q}})\right),\\
\mathcal{V}_\perp(\alpha_i) &= -30\alpha_g^2\left[h_{00}(1-\alpha_g) + h_{01}(\alpha_g(1-\alpha_g) - 6\alpha_q\alpha_{\bar{q}}) + h_{10}(\alpha_g(1-\alpha_g) - \frac{3}{2}(\alpha_{\bar{q}}^2 + \alpha_q^2))\right],\\
\mathcal{A}_\perp(\alpha_i) &= 30\alpha_g^2(\alpha_{\bar{q}} - \alpha_q)\left[h_{00} + h_{01}\alpha_g + \frac{1}{2}h_{10}(5\alpha_g - 3)\right],\\
B(u) &= g_\pi(u) - \phi_\pi(u),\\
g_\pi(u) &= g_0 C_0^{\frac{1}{2}}(2u-1) + g_2 C_2^{\frac{1}{2}}(2u-1) + g_4 C_4^{\frac{1}{2}}(2u-1),\\
\mathbb{A}(u) &= 6u\bar{u}\left[\frac{16}{15} + \frac{24}{35}a_2^\pi + 20\eta_3 + \frac{20}{9}\eta_4 + \left(-\frac{1}{15} + \frac{1}{16} - \frac{7}{27}\eta_3 w_3 - \frac{10}{27}\eta_4\right)C_2^{\frac{3}{2}}(2u-1)\right.\\
&+ \left.\left(-\frac{11}{210}a_2^\pi - \frac{4}{135}\eta_3 w_3\right)C_4^{\frac{3}{2}}(2u-1)\right]\\
&+ \left(-\frac{18}{5}a_2^\pi + 21\eta_4 w_4\right)\left[2u^3(10 - 15u + 6u^2)\ln u\right.\\
&+ \left. 2\bar{u}^3(10 - 15\bar{u} + 6\bar{u}^2)\ln\bar{u} + u\bar{u}(2 + 13u\bar{u})\right],
\end{aligned} \tag{17}$$

where $C_n^k(x)$ are the Gegenbauer polynomials,

$$\begin{aligned}
h_{00} &= v_{00} = -\frac{1}{3}\eta_4, \\
a_{10} &= \frac{21}{8}\eta_4 w_4 - \frac{9}{20}a_2^\pi, \\
v_{10} &= \frac{21}{8}\eta_4 w_4, \\
h_{01} &= \frac{7}{4}\eta_4 w_4 - \frac{3}{20}a_2^\pi, \\
h_{10} &= \frac{7}{4}\eta_4 w_4 + \frac{3}{20}a_2^\pi, \\
g_0 &= 1, \\
g_2 &= 1 + \frac{18}{7}a_2^\pi + 60\eta_3 + \frac{20}{3}\eta_4, \\
g_4 &= -\frac{9}{28}a_2^\pi - 6\eta_3 w_3.
\end{aligned} \tag{18}$$

The constants in the Eqs. (17) and (18) were calculated at the renormalization scale $\mu = 1 \ \ GeV^2$ using QCD sum rules [24–26] and are given as $a_1^\pi = 0$, $a_2^\pi = 0.44$, $\eta_3 = 0.015$, $\eta_4 = 10$, $w_3 = -3$ and $w_4 = 0.2$.

The sum rules for the coupling constants contain three auxiliary parameters, the Borel mass parameter M^2, the continuum threshold s_0 and the general parameter β. These are not physical parameters, hence the sum rules for the coupling constants should be independent of them. Therefore, we should look for the working regions for these parameters such that the dependency on these parameters are weak. At too large values of the Borel parameter M^2, the suppression of the contribution of higher states and continuum is reduced, increasing the error due to the quark-hadron duality approximation. The lower limit of the M^2 can be obtained by requiring that the highest twist contribution, which is suppressed by a higher power of M^2, should be small. Therefore, the higher states and continuum contributions should comprise a small percentage of the total dispersion integral. The continuum threshold s_0 is not completely arbitrary and is related to the energy of the first exited state. From the numerical analysis of the sum rules for the coupling constants, the continuum thresholds s_0 are obtained as $38 \ GeV^2 \leq s_0 \leq 42 \ GeV^2$ and $9 \ GeV^2 \leq s_0 \leq 11 \ GeV^2$ for the $\Sigma_b\Sigma_b\pi$ and $\Sigma_c\Sigma_c\pi$ vertexes, respectively. In order to acquire the working region for β parameter, we plot the $g_{\Sigma_Q\Sigma_Q\pi}$ as a function of $\cos\theta$ in the interval $-1 \leq \cos\theta \leq 1$, which corresponds to $-\infty \leq \beta \leq \infty$, where $\tan\theta = \beta$. From the Figs. (1) and (2), which show the dependency of coupling constants on the $\cos\theta$, it is seen that for $-0.5 \leq \cos\theta \leq 0.4$ the dependence of the coupling constants on $\cos\theta$ is weak. We present also the dependency of the coupling constants, $g_{\Sigma_c\Sigma_c\pi}$ and $g_{\Sigma_b\Sigma_b\pi}$, on the Borel parameter M^2 in Figs. (3) and (4), respectively. These Figures reveal that the coupling constants are quite stable in the chosen Borel mass region. From these Figures, the numerical values of the coupling constants, $g_{\Sigma_c\Sigma_c\pi}$ and $g_{\Sigma_b\Sigma_b\pi}$, can be extracted as follows:

$$g_{\Sigma_c\Sigma_c\pi} = -8.0 \pm 1.7, \qquad\qquad g_{\Sigma_b\Sigma_b\pi} = -11.0 \pm 2.1. \tag{19}$$

The errors appearing in our predictions are due to the uncertainties in the input as well as the auxiliary parameters.

At the end of this section, let us compare our results with the existing prediction on the $g_{\Sigma^+\Sigma^0\pi^+}$ coupling constant [22] for the light Σ baryon. In light case the s quark and in our case (baryons with a single heavy quark) the c or b quark is the spectator quark. These quarks do not participate in the considered strong interactions, so one expects these coupling constants (heavy and light cases) have values close to each other. In [22], it was estimated that, $g_{\Sigma^+\Sigma^0\pi^+} = -9 \pm 2$. The same transition for the light Σ baryon was also studied in [28] and [29]. In [28], the ratio, $g_{\Sigma^+\Sigma^0\pi^+}/g_{NN\pi} \simeq 0.8$ was obtained, which leads to the value of the coupling constant, $g_{\Sigma^+\Sigma^0\pi^+} \simeq 11.9$ with the experimental value of $g_{NN\pi} = 14.9$ [30]. The value of the same coupling constant was given in [29] as $g_{\Sigma^+\Sigma^0\pi^+} = -11.9 \pm 0.4$. Comparison of our results with that of [22, 28, 29] supports the expectation that the spectator quark does not effect the pionic coupling, significantly.

IV. ACKNOWLEDGMENT

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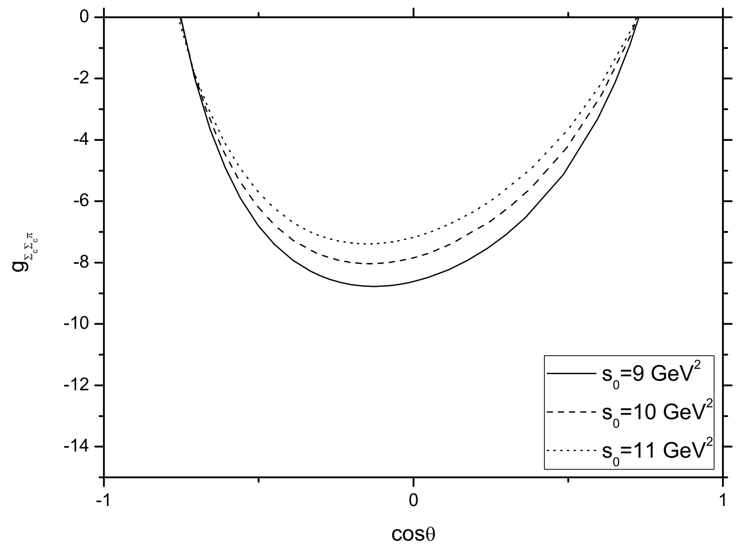

FIG. 1: The dependence of the coupling constant $g_{\Sigma_c\Sigma_c\pi}$ on $\cos\theta$ for the value of Borel parameter $M^2 = 7$ GeV^2 and the different values of continuum threshold s_0.

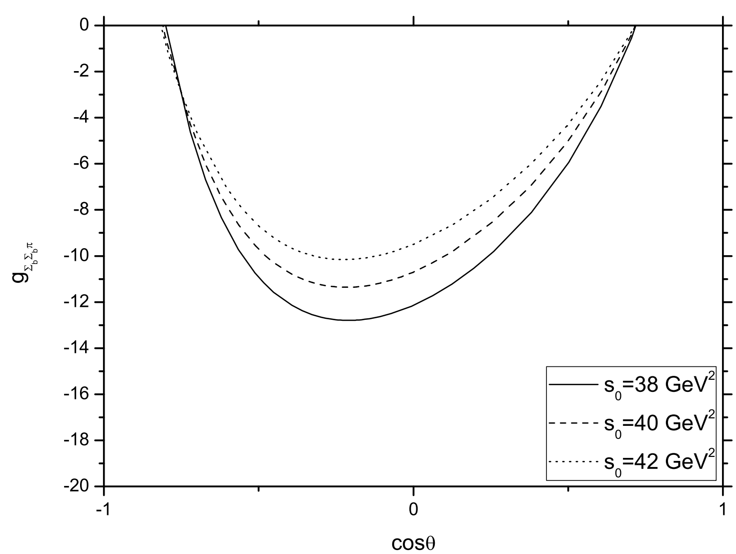

FIG. 2: The dependence of the coupling constant $g_{\Sigma_b\Sigma_b\pi}$ on $\cos\theta$ for the value of Borel parameter $M^2 = 30$ GeV^2 and the different values of continuum threshold s_0.

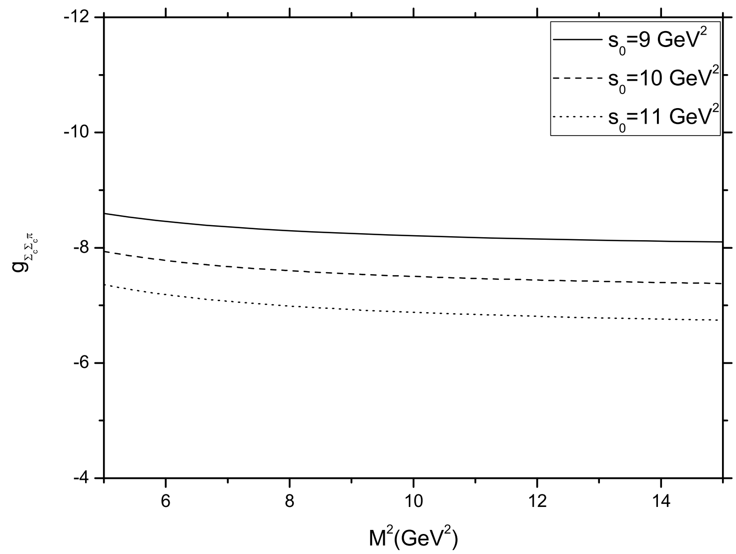

FIG. 3: The dependence of the coupling constant $g_{\Sigma_c \Sigma_c \pi}$ on the Borel parameter M^2 for the value of arbitrary parameter $\beta = -3$ and the different values of the continuum threshold s_0.

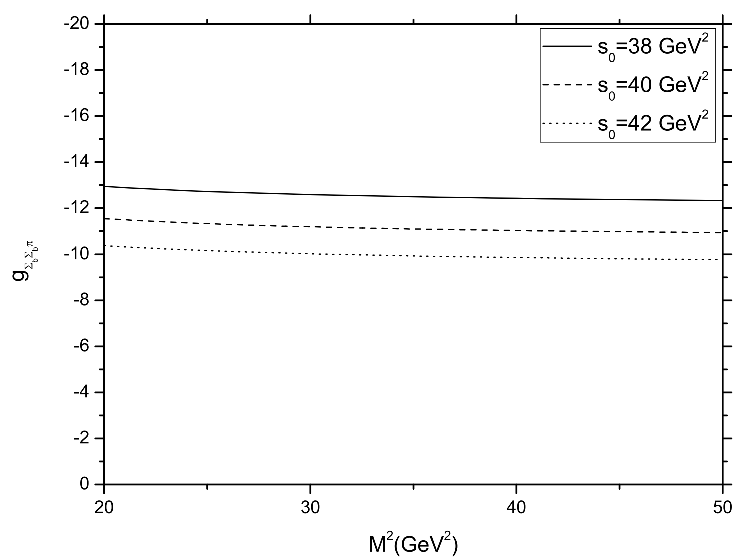

FIG. 4: The dependence of the coupling constant $g_{\Sigma_b \Sigma_b \pi}$ on the Borel parameter M^2 for the value of arbitrary parameter $\beta = -3$ and the different values of the continuum threshold s_0.